LESSON 11-3

Practice A

Exponential Growth and Decay

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales for a clothing store are $270,000 and are increasing at a rate of 7% per year; 3 years
   $y = \boxed{270{,}000}(1 + \boxed{0.07})^{\boxed{3}}$
   $y = \$330{,}761.61$

2. The population of a school is 2200 and is increasing at a rate of 2%; 6 years
   $y = \boxed{2200}(1 + \boxed{0.02})^{\boxed{6}}$
   $y \approx 2478$

3. The value of an antique vase is $200 and is increasing at a rate of 8%; 12 years
   $y = 200(1 + 0.08)^{12}$
   $y \approx \$503.63$

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $20,000 invested at a rate of 3% compounded annually; 8 years.
   $A = \boxed{20{,}000}\left(1 + \frac{\boxed{0.03}}{\boxed{1}}\right)^{(\boxed{1})(\boxed{8})}$
   $A \approx \$25{,}335.40$

5. $35,000 invested at a rate of 6% compounded monthly; 10 years
   $A = \boxed{35{,}000}\left(1 + \frac{\boxed{0.06}}{\boxed{12}}\right)^{(\boxed{12})(\boxed{10})}$
   $A \approx \$63{,}678.89$

6. $35,000 invested at a rate of 8% compounded quarterly; 5 years
   $A = 35{,}000\left(1 + \frac{0.08}{4}\right)^{20}$
   $A \approx \$52{,}008.16$

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

7. The population of a school is 800 and is decreasing at a rate of 2% per year; 4 years
   $y = \boxed{800}(1 - \boxed{0.02})^{\boxed{4}}$
   $y \approx 738$

8. The bird population in a forest is about 2300 and decreasing at a rate of 4% per year; 10 years
   $y = \boxed{2300}(1 - \boxed{0.04})^{\boxed{10}}$
   $y \approx 1529$

9. The half-life of strontium-90 is approximately 28 years. Find the amount of strontium-90 left from a 10 gram sample after 56 years.
   $A = \boxed{10}(0.5)^{\boxed{2}}$
   $A \approx 2.5$ grams

LESSON 11-3

Practice B

Exponential Growth and Decay

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales for a fast food restaurant are $650,000 and are increasing at a rate of 4% per year; 5 years
   $y = 650{,}000(1.04)^x$
   $\approx \$790{,}824.39$

2. The population of a school is 800 students and is increasing at a rate of 2% per year; 6 years
   $y = 800(1.02)^x$
   ≈ 901

3. During a certain period of time, about 70 northern sea otters had an annual growth rate of 18%; 4 years
   $y = 70(1.18)^x$
   ≈ 136

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $50,000 invested at a rate of 3% compounded monthly; 6 years
   $y = 50{,}000(1.0025)^{12t}$
   $\approx \$59{,}847.42$

5. $43,000 invested at a rate of 5% compounded annually; 3 years
   $y = 43{,}000(1.05)^t$
   $\approx \$49{,}777.88$

6. $65,000 invested at a rate of 6% compounded quarterly; 12 years
   $y = 65{,}000(1.015)^{4t}$
   $\approx \$132{,}826.09$

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

7. The population of a town is 2500 and is decreasing at a rate of 3% per year; 5 years
   $y = 2500(0.97)^x$
   ≈ 2147

8. The value of a company's equipment is $25,000 and decreases at a rate of 15% per year; 8 years
   $y = 25{,}000(0.85)^x$
   $\approx \$6812.26$

9. The half-life of Iodine-131 is approximately 8 days. Find the amount of Iodine-131 left from a 35 gram sample after 32 days.
   2.1875 grams

LESSON 11-3

Practice C

Exponential Growth and Decay

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales for a furniture store are $375,000 and are increasing at a rate of 6.75% each year; 9 years
   $y = 375{,}000(1.0675)^x$
   $\approx \$675{,}059.76$

2. The population of Indiana showed an annual growth rate of 0.6% between 1998 and 1999. The population in 1999 was approximately 273,000,000. Based on this model, find the population in 2007.
   $y = 273{,}000{,}000(1.006)^x$
   $\approx 286{,}382{,}511$

3. Per capita income is the total income for a geographic area divided by the number of people in that area. In Florida, the per capita personal income (PCPI) of $30,098 is increasing at a rate of 3.6%; 8 years
   $y = 30{,}098(1.036)^x$
   $\approx \$39{,}940.70$

Write a compound interest function to model each situation. Then find the balance after the given number of years.

4. $60,000 invested at a rate of 2.5% compounded annually; 8 years
   $y = 60{,}000(1.025)^t$
   $\approx \$73{,}104.17$

5. $27,000 invested at a rate of 3.75% compounded quarterly; 3 years
   $y = 27{,}000(1.009375)^{4t}$
   $\approx \$30{,}199.12$

6. $95,000 invested at a rate of 4.2% compounded monthly; 10 years
   $y = 95{,}000(1.0035)^{12t}$
   $\approx \$144{,}480.36$

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

7. The population of a town is 1800 and is decreasing at a rate of 3.8% per year; 6 years
   $y = 1800(0.962)^x$
   ≈ 1427

8. A population of 2300 manatees in Florida is thought to be decreasing at a rate of 1.1% annually; 7 years
   $y = 2300(0.989)^x$
   ≈ 2129

9. The half-life of Cobalt-60 is approximately 5.25 days. Find the amount of Cobalt-60 left from a 30 gram sample after 42 days. Round to the nearest thousandth of a gram.
   ≈ 0.117 grams

LESSON 11-3

Reteach

Exponential Growth and Decay

In the exponential growth and decay formulas, y = final amount, a = original amount, r = rate of growth or decay, and t = time.

Exponential growth: $y = a(1 + r)^t$

The population of a city is increasing at a rate of 4% each year. In 2000 there were 236,000 people in the city. Write an exponential growth function to model this situation. Then find the population in 2009.

Step 1: Identify the variables.

$a = 236{,}000 \quad r = 0.04$

Step 2: Substitute for a and r.

$y = a(1 + r)^t$

$y = 236{,}000(1 + 0.04)^t$

The exponential growth function is $y = 236{,}000(1.04)^t$.

Growth = greater than 1.

Step 3: Substitute for t.

$y = 236{,}000(1.04)^9$

$\approx 335{,}902$

The population will be about 335,902.

Exponential decay: $y = a(1 - r)^t$

The population of a city is decreasing at a rate of 6% each year. In 2000 there were 35,000 people in the city. Write an exponential decay function to model this situation. Then find the population in 2012.

Step 1: Identify the variables.

$a = 35{,}000 \quad r = 0.06$

Step 2: Substitute for a and r.

$y = a(1 - r)^t$

$y = 35{,}000(1 - 0.06)^t$

The exponential decay function is $y = 35{,}000(0.94)^t$.

Decay = less than 1.

Step 3: Substitute for t.

$y = 35{,}000(0.94)^{12}$

$\approx 16{,}657$

The population will be about 16,657.

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

1. Annual sales at a company are $372,000 and increasing at a rate of 5% per year; 8 years
   $y = \boxed{372{,}000}(1 + \boxed{0.05})^{\boxed{8}}$
   $\approx \$549{,}613$

2. The population of a town is 4200 and increasing at a rate of 3% per year; 7 years
   $y = 4200(1.03)^7$; ≈ 5165

Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

3. Monthly car sales for a certain type of car are $350,000 and are decreasing at a rate of 3% per month; 6 months
   $y = \boxed{350{,}000}(1 - \boxed{0.03})^{\boxed{6}}$;
   $\approx \$291{,}540$

4. An internet chat room has 1200 participants and is decreasing at a rate of 2% per year; 5 years
   $y = 1200(0.98)^5$; ≈ 1085

LESSON 11-3

Reteach

Exponential Growth and Decay (continued)

A special type of exponential growth involves finding compound interest.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

- where A is the total balance after t years
- P is the original amount
- r is the interest rate
- n is the number of times the interest is compounded in one year
- t is the number of years

Write a compound interest function to model \$15,000 invested at a rate of 3% compounded quarterly. Then find the balance after 8 years.

$A = 15{,}000\left(1 + \frac{0.03}{4}\right)^{4t}$

$A = 15{,}000(1.0075)^{4t}$ *Compound interest function*

$A = 15{,}000(1.0075)^{4(8)}$ *Substitute 8 for t.*

$A = 15{,}000(1.0075)^{32}$

$\approx 19{,}051.67$

The balance after 8 years is \$19,051.67.

A special type of exponential decay involves the half-life of substances.

$A = P(0.5)^t$

- where A is the final amount
- P is the original amount
- t is the number of half-lives in a given time period

Ismuth-212 has a half-life of approximately 60 seconds. Find the amount of Ismuth-212 left from a 25 gram sample after 300 seconds.

Step 1: Find t. $t = \frac{300}{60} = 5$

Step 2: Substitute for P and t.

$A = 25(0.5)^5$

$= 0.78125$

The amount after 300 s is 0.78125 g.

Write a compound interest function to model each situation. Then find the balance after the given number of years.

5. \$17,000 invested at 3%, compounded annually; 6 years
   $A = 17{,}000(1.03)^t$
   \$20,298.89

6. \$23,000 invested at 2%, compounded quarterly; 8 years
   $A = 23{,}000\left(1 + \frac{0.02}{4}\right)^{4t}$
   \$26,979.99

Write an exponential decay function to model each situation. Then find the value after the given amount of time.

7. A 30 gram sample of Iodine-131 has a half-life of about 8 days; 24 days
   3.75 g

8. A 40 gram sample of Sodium-24 has a half-life of 15 hours; 60 hours
   2.5 g

LESSON 11-3

Challenge

Planning to Make Donations

Some of the parents of the students in a mathematics class invest money in accounts that pay compound interest. Their hope is to have their investments grow so that, at some time in the future, they can use the money that has accumulated to help the local Boys and Girls Club. The compound interest that they need for their financial planning is shown at right.

$A = P(1 + r)^t =$
- P represents the initial amount.
- r represents the rate as a decimal
- t represents time in years
- A represents the final amount

Suppose that \$10,000 is invested into an account that pays 5.65% interest compounded annually. Give answers rounded to the nearest cent.

1. Write a function that will give A as a function of t. $A = 10{,}000(1.0565)^t$

2. Complete the table below.

t	0	1	2	3	4	5	6	7
A	10,000	10,565	11,161.92	11,792.57	12,458.85	13,162.78	13,906.47	14,692.19

3. a. When will their initial deposit exceed \$12,000?
   sometime after the end of the third year but before the end of the fourth year

   b. When will their initial deposit exceed \$12,000 but be less than \$14,000?
   sometime after the end of the third year but before the end of the fourth year and sometime after the end of the sixth year but before the end of the seventh year

4. The group of parents want their initial deposit amount to grow to \$15,000. Use guess-and-check to approximate the number of years that it will take for this to happen. Give your answer to the nearest tenth of a year. 7.4 years

5. The group wants their initial deposit amount to grow to \$18,000 within 10 years. After that time, they want to make an \$18,000 donation to the local club. Will they achieve their goal? Explain your response.
   No; after 10 years, they will have \$17,325.87, which is less than \$18,000.

Use guess-and-check to approximate the time that it takes the given investment to grow to the target goal. Round answers to the nearest tenth of a year.

6. \$12,500 at 4.5% compounded yearly; target goal: \$18,000 — 8.3 years
7. \$15,500 at 6.5% compounded yearly; target goal: \$20,000 — 4.1 years
8. \$5 at 5.5% compounded yearly; target goal: \$18,000 — 153 years

LESSON 11-3

Problem Solving

Exponential Growth and Decay

Write the correct answer.

1. A condo in Austin, Texas, was worth \$80,000 in 1990. The value of the condo increased by an average of 3% each year. Write an exponential growth function to model this situation. Then find the value of the condominium in 2005.
   $y = 80{,}000(1.03)^t$
   \$124,637

2. Markiya deposited \$500 in a savings account. The annual interest rate is 2%, and the interest is compounded monthly. Write a compound interest function to model this situation. Then find the balance in Markiya's account after 4 years.
   $y = 500\left(1 + \frac{0.02}{12}\right)^{12t}$
   \$541.61

3. The population of a small Midwestern town is 4500. The population is decreasing at a rate of 1.5% per year. Write an exponential decay function to model this situation. Then find the number of people in the town after 25 years.
   $y = 4500(0.985)^t$
   3084

4. Twelve students at a particular high school passed an advanced placement test in 2000. The number of students who passed the test increased by 16.4% each year thereafter. Find the number of students who passed the test in 2004.
   22

Half-lives range from less than a second to billions of years. The table below shows the half-lives of several substances. Select the best answer.

Half-Lives	
Nitrogen-16	7 s
Technetium-99	6 h
Sulfur-35	87 days
Tritium	12.3 yr
Uranium-238	4.5 billion yrs

5. About how many grams of a 500 g sample of Technetium-99 is left after 2 days?
   (A) 1.95 g　**C** 31.25 g
   B 7.81 g　**D** 62.5 g

6. Which equation can be used to find how much of a 50 g sample of Nitrogen-16 is left after 7 minutes?
   F $A = 50(0.5)^1$　**H** $A = 50(0.5)^{42}$
   G $A = 50(0.5)^7$　**(J)** $A = 50(0.5)^{60}$

7. How many billions of years will it take 1000 grams of Uranium-238 to decay to just 125 grams?
   A 0.125　**C** 9
   B 3　**(D)** 13.5

8. A researcher had 37.5 g left from a 600 g sample of Sulfer-35. How many half-lives passed during that time?
   (F) 4　**H** 7
   G 5　**J** 16

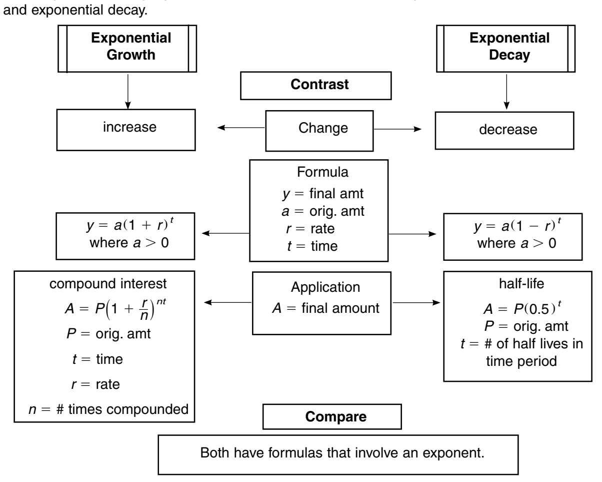
9. Look at problem 8. How many days passed during that time?
   A 7　**(C)** 348
   B 16　**D** 435

LESSON 11-3

Reading Strategies

Compare and Contrast

The diagram below highlights important concepts of exponential growth and exponential decay.



For each situation: a. identify it as exponential growth or exponential decay, and b. use a formula to calculate the answer.

1a. The bird population in an wooded area is decreasing by 3% each year from 1250.
   exponential decay

1b. Find the bird population after 6 years.
   1041 birds

2a. A town's population was 3800 in 2005 and growing at a rate of 2% every year.
   exponential growth

2b. Find the town's population in 2025.
   5647 people

3a. \$800 is invested at a rate of 4% and is compounded monthly (12 times/year).
   exponential growth

3b. Find the balance after 10 years.
   \$1192.67